AMS/ECON 11B **Quiz 2- Solutions** **October 14, 2015**

(1) Compute the ***indefinite*** integrals...

a. (2 pts) $\displaystyle\int \frac{3x}{\sqrt{2x^2+7}}\,dx = 3\int \frac{x}{\sqrt{2x^2+7}}\,dx = \ldots$

Substitute $u = 2x^2+7 \implies du = 4x\,dx \implies x\,dx = \frac{1}{4}\,du$, so

$$3\int \frac{x}{\sqrt{2x^2+7}}\,dx = 3\int \frac{1}{4}u^{-1/2}\,du = \frac{3}{4}\cdot\frac{u^{1/2}}{1/2} + C = \boxed{\frac{3}{2}(2x^2+7)^{1/2} + C}$$

b. (2 pts) $\displaystyle\int \frac{(\ln x+3)^2}{x}\,dx = \int (\ln x+3)^2\cdot\frac{1}{x}\,dx = \ldots$

Substitute $u = \ln x + 3 \implies du = \frac{1}{x}\,dx$, so

$$\int (\ln x+3)^2\cdot\frac{1}{x}\,dx = \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{1}{3}(\ln x+3)^3 + C}$$

(2) Compute the ***definite*** integrals...

a. (3 pts) $\displaystyle\int_1^3 3x^2+4x+5\,dx = \left. x^3+2x^2+5x\right|_1^3 = (27+18+15)-(1+2+5) = 52$

b. (3 pts) $\displaystyle\int_0^3 \frac{5x}{x^2+1}\,dx = 5\int_0^3 \frac{x}{x^2+1}\,dx = \ldots$

Substitute $u = x^2 + 1 \implies du = 2x\,dx \implies x\,dx = \frac{1}{2}\,du$.

Also, limits of integration change accordingly:

$x = 0 \implies u = 0^2 + 1 = 1$ and $x = 3 \implies u = 3^2 + 1 = 10$, so

$$5\int_0^3 \frac{x}{x^2+1}\,dx = \frac{5}{2}\int_1^{10} \frac{1}{u}\,du = \frac{5}{2}\ln|u|\Big|_1^{10} = \frac{5}{2}\ln 10 - \frac{5}{2}\ln 1 = \frac{5}{2}\ln 10 \quad (\approx 5.756).$$

Alternatively, find the antiderivative first (using the same substitution in the corresponding ***in***definite integral) and then use the fundamental theorem of calculus:

$$5\int \frac{x}{x^2+1}\,dx = \frac{5}{2}\int \frac{1}{u}\,du = \frac{5}{2}\ln|u| + C = \frac{5}{2}\ln(x^2+1) + C, \text{ so}$$

$$5\int_0^3 \frac{x}{x^2+1}\,dx = \frac{5}{2}\ln(x^2+1)\Big|_1^3 = \frac{5}{2}\ln 10 - \frac{5}{2}\ln 1 = \frac{5}{2}\ln 10$$