AMS/ECON 11B Quiz 5–Solutions November 18, 2015

1. (6 pts) Find the critical point(s) and critical value(s) of the function

$$f(x,y) = \frac{1}{3}x^3 + 4xy + 2y^2 - 5x + 2.$$

First order conditions:

$$f_x = 0 \implies x^2 + 4y - 5 = 0 \implies x^2 - 4x - 5 = 0 \implies \boxed{\boxed{(x-5)(x+1) = 0}}$$

$$\searrow \qquad \uparrow$$

$$f_y = 0 \implies 4x + 4y = 0 \implies \boxed{4y = -4x} \implies \boxed{\boxed{y = -x}}$$

The critical x-values are $x_1 = 5$ and $x_2 = -1$, the critical points are

$$(x_1, y_1) = (5, -5) \text{ and } (x_2, y_2) = (-1, 1),$$

and the critical values of the function are

$$f(5,-5) = -\frac{94}{3} \text{ and } f(-1,1) = \frac{14}{3}.$$

2. (4 pts) Use the second derivative test to classify the critical value(s) of the function above as relative minima, relative maxima or neither.

Second derivatives:

$$f_{xx} = 2x, \; f_{xy} = 4 \;\text{ and }\; f_{yy} = 4.$$

Discriminant:

$$D(x, y) = f_{xx}(x, y) f_{yy}(x, y) - (f_{xy}(x, y))^2 = 8x - 16.$$

At the critical point $(5, -5)$, we have

$$D(5, -5) = 40 - 16 = 24 > 0 \;\text{ and }\; f_{xx}(5, -5) = 10 > 0,$$

so $f(5, -5) = -94/3$ is a ***local minimum*** value.

At the critical point $(-1, 1)$

$$D(-1, 5) = -8 - 16 = -24 < 0,$$

so $f(-1, 1)$ is ***not*** a local extreme value ($(-1, 1, 14/3)$ is a saddle point on the graph $z = f(x, y)$).