AMS/ECON 11B   Quiz 6 − Solutions   November 23, 2015

1. (10 pts) A monopolistic firm sells one product in two markets, A and B. The daily demand equation for the firm's product in each market are given by

$$Q_A = 100 - 0.4P_A \qquad \text{and} \qquad Q_B = 120 - 0.5P_B,$$

where Q_A and Q_B are the demands and P_A and P_B are the prices for the firm's product in each market, respectively. The firm's cost function is given by $C = 40(Q_A + Q_B) + 2500$.

Find the prices that the firm should charge in each market to *maximize its daily profit*. What is the firm's max profit? Use the second derivative test to verify that the prices you found yield the maximum profit.

1. Profit = revenue − cost, and revenue = price × quantity, so the firm's profit function in this problem is

$$\Pi = P_A Q_A + P_B Q_B - C$$
$$= P_A(100 - 0.4P_A) + P_B(120 - 0.5P_B) - (40(100 - 0.4P_A + 120 - 0.5P_B) + 2500)$$
$$= -0.4P_A^2 + 116PA - 0.5P_B^2 + 140P_B - 11300$$

2. To find critical point(s), we solve the system $\Pi_{P_A} = 0$ and $\Pi_{P_B} = 0$:

$$\Pi_{P_A} = 0 \implies -0.8P_A + 116 = 0 \qquad \implies \qquad P_A^* = \frac{116}{0.8} = 145$$
$$\Pi_{P_B} = 0 \implies -P_B + 140 = 0 \qquad \implies \qquad P_B^* = 140$$

3. Second derivative test: $\Pi_{P_A P_A} = -0.8$, $\Pi_{P_A P_B} = 0$ and $\Pi_{P_B P_B} = -1$, so the discriminant is

$$D = \Pi_{P_A P_A} \Pi_{P_B P_B} - \Pi^2_{P_A P_B} = 0.8 > 0$$

and $\Pi_{P_A P_A} = -0.8 < 0$, which implies that $\Pi^* = \Pi(145, 140) = 6910$ is the max profit, as required.