Final Exam

Instructions

- Please turn off all phones and other electronic devices.
- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided.
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Read the questions carefully and check your answers.***
- ***For full credit—show all your work.***

Good Luck!!!

NAME: ____

Problem	Score
1	/8
2	/8
3	/10
4	/8
5	/10
6	/10
Total	/50

Selected Integration Formulas

Basic rules.

1. $\displaystyle\int u^k\,du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C.$

3. $\displaystyle\int e^u\,du = e^u + C.$

4. $\displaystyle\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\displaystyle\int c\cdot f(u)\,du = c\cdot\int f(u)\,du.$

Rational forms containing $(\mathbf{a}+\mathbf{bu})$*.*

6. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C.$

7. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C.$

8. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C.$

9. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a}+\mathbf{bu}}$*.*

10. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle\int e^{au}\,du = \frac{e^{au}}{a} + C.$

14. $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C.$

15. $\displaystyle\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}\ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (8 pts) Compute the *present value* of a continuous annuity that pays at the annual rate $f(t) = 1000t$ for $T = 10$ years, assuming that interest is compounded continuously at the rate $r = 5.5\%$.

2. (8 pts) Find the *Consumers' surplus* and *Producers' surplus* at equilibrium for the market whose supply and demand equations are given below.

- *Supply*: $p = 5 + 0.125q$,
- *Demand*: $p = 90 - 0.05q^2$.

3. The average monthly demand (Q) for a ACME Widgets' product is related to the price of their Widgets (p), the average price of substitutes for Widgets (p_s) and the average monthly household income in the market for the firm's product (Y), by the equation

$$Q = \frac{100(2Y + 15p_s - 1650)^{3/5}}{2p + 10},$$

where Q is measured in 1000s of Widgets, and the prices and income are all measured in dollars.

a. (6 pts) Compute Q_p, Q_{p_s} Q_Y when $p = 20$, $p_s = 25$ and $Y = 2200$.

b. (2 pts) Compute the *income-elasticity of demand* when $p = 20$, $p_s = 25$ and $Y = 2200$.

c. (2 pts) Suppose that income remains fixed and both prices *increase* by \$1. Use your answer to **a.** to estimate the change in demand for ACME's product.

Round your answers to 2 decimal places.

4. (8 pts) Find the critical points of the function

$$f(x, y) = x^3 + 2x^2 + 2xy - y^2 - 9y + 1$$

and classify the critical values using the second derivative test.

5. Industrial Gadget's (IG) production function is given by

$$Q = 30K^{2/3}L^{1/3},$$

where Q is the firm's annual output, measured in gadgets, K is the firm's monthly capital input and L is the firm's monthly labor input. The price per unit of capital is $p_K = \$5,000$ and the price per unit of labor is $p_L = \$3,000$.

a. (6 pts) Find the levels of capital and labor input that IG should use to ***minimize the cost*** of producing $Q_0 = 10,000$ gadgets. What is the minimum cost?

b. (2 pts) What is *IG*'s *marginal cost* at that level of production? Explain your answer.

c. (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in *IG*'s (minimum) cost of producing 10,000 gadgets, if the price per unit of capital increases to \$5,100 (assuming that all else stays the same).

6. The Smith family's utility function is given by

$$U(x, y, z) = 7\ln x + 8\ln y + 10\ln z,$$

where x, y and z are the quantities of *X-wares, Y-wares* and *Z-wares* that they consume per month. The average prices of these wares are $p_x = \$10$, $p_y = \$15$ and $p_z = \$20$, respectively.

a. (8 pts) Find the quantities of X-wares, Y-wares and Z-wares that the Smith family should consume each month to maximize their utility, given that their monthly XYZ-budget is $B = \$4500$. What is their maximum utility?

b. (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in the Smith's monthly utility if the price of Z-wares *decreases* to \$19.50 (assuming that all else stays the same).