Final Exam

Instructions

- Please turn off all phones and other electronic devices.
- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided.
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Read the questions carefully and check your answers.***
- ***For full credit—show all your work.***

Good Luck!!!

NAME: _____

Problem	Score
1	/8
2	/8
3	/8
4	/10
5	/10
6	/10
Total	/50

Selected Integration Formulas

Basic rules.

1. $\displaystyle\int u^k\,du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C.$

3. $\displaystyle\int e^u\,du = e^u + C.$

4. $\displaystyle\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\displaystyle\int c \cdot f(u)\,du = c \cdot \int f(u)\,du.$

Rational forms containing $(\mathbf{a + bu})$*.*

6. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C.$

7. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C.$

8. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C.$

9. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a + bu}}$*.*

10. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle\int e^{au}\,du = \frac{e^{au}}{a} + C.$

14. $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C.$

15. $\displaystyle\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}\ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (8 pts) A firm's *marginal cost* function is given by

$$\frac{dc}{dq} = \frac{5q + 12}{q + 2}$$

and the firm's fixed cost is \$3000. Find the firm's cost function.

2. (8 pts) Find the *Consumers' surplus* and *Producers' surplus* at equilibrium for the market whose supply and demand equations are given below.

- *Supply*: $p = 10 + 0.25q$,
- *Demand*: $p = 180 - 0.1q^2$.

3. The average monthly demand for a ACME Widgets' product is given by the relation

$$Q = 4\ln(5Y^2 - 3p),$$

where Q is the monthly demand, measured in 1000s of widgets; p is price of an ACME Widget, measured in dollars; and Y is the average monthly household income in the market for the firm's product, measured in \$1000s.

(a) (5 pts) Find Q, Q_p and Q_Y when $p = 25$ and $Y = 4$.

(b) (3 pts) Find the *price-elasticity of demand* when $p = 25$ and $Y = 4$.

Round your answers to 2 decimal places.

4. (10 pts) A monopolistic firm produces two types of widgets, type A and type B widgets. It costs the firm \$8 per unit to produce type A widgets and \$4 per unit to produce type B widgets, and their weekly fixed cost is \$2000. The demand functions for these products are given by

$$\begin{aligned} Q_A &= 400(P_B - 2P_A + 14) \\ Q_B &= 400(P_A - P_B), \end{aligned}$$

where P_A and P_B are the selling prices per unit of type A and type B widgets, and Q_A and Q_B are the numbers of type A and type B widgets that the firm sells in a week.

Find the prices that the firm should set to maximize their weekly profit. Use the second derivative test to justify your claim that the firm's profit is maximized at the prices you found.

5. ACME Widget's (AW) production function is given by

$$Q = 10K^{0.6}L^{0.4},$$

where Q is the firm's annual output, measured in widgets, K is the firm's annual capital input and L is the firm's annual labor input. The price per unit of capital is $p_K = \$5,000$ and the price per unit of labor is $p_L = \$2,000$.

(a) (6 pts) Find the levels of capital and labor input that AW should use to ***minimize the cost*** of producing $Q_0 = 4,000$ widgets. What is the minimum cost?

(b) (2 pts) What is AW's ***marginal cost***? Explain your answer.

(c) (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in AW's (minimum) cost of producing 4,000 widgets, if the price per unit of capital increases to \$5,200 (assuming that all else stays the same).

6. The Smith family's utility function is given by

$$U(x, y, z) = 12 \ln x + 7 \ln y + 5 \ln z,$$

where x, y and z are the quantities of *X-goods, Y-goods* and *Z-goods* that they consume per month. The average prices of these goods are $p_x = \$15$, $p_y = \$10$ and $p_z = \$5$, respectively.

(a) (7 pts) Find the quantities of X-goods, Y-goods and Z-goods that the Smith family should consume each month to maximize their utility, given that their monthly XYZ-budget is $B = \$3600$. What is their maximum utility?

(b) (3 pts) By approximately how much will the Smiths need to increase their XYZ-budget if they want to increase their monthly utility by 3 *utils*? Explain your answer.