Final Exam

Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided.
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Take your time. Answer each question completely. Check your answers.***
- ***For full credit—explain/show your work.***

Good Luck!!!

NAME: ______________________________

Problem	Score
1	/9
2	/9
3	/9
4	/9
5	/9
6	/9
Total	/50

Selected Integration Formulas

Basic rules.

1. $\int u^k\,du = \dfrac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\int \dfrac{1}{u}\,du = \ln|u| + C.$

3. $\int e^u\,du = e^u + C.$

4. $\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\int c \cdot f(u)\,du = c \cdot \int f(u)\,du.$

Rational forms containing $(\mathbf{a+bu})$*.*

6. $\int \dfrac{du}{a+bu} = \dfrac{1}{b}\ln|a+bu| + C.$

7. $\int \dfrac{u\,du}{a+bu} = \dfrac{u}{b} - \dfrac{a}{b^2}\ln|a+bu| + C.$

8. $\int \dfrac{u^2\,du}{a+bu} = \dfrac{u^2}{2b} - \dfrac{au}{b^2} + \dfrac{a^2}{b^3}\ln|a+bu| + C.$

9. $\int \dfrac{u^2\,du}{(a+bu)^2} = \dfrac{u}{b^2} - \dfrac{a^2}{b^3(a+bu)} - \dfrac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a+bu}}$*.*

10. $\int u\sqrt{a+bu}\,du = \dfrac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\int \dfrac{u\,du}{\sqrt{a+bu}} = \dfrac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\int \dfrac{u^2\,du}{\sqrt{a+bu}} = \dfrac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\int e^{au}\,du = \dfrac{e^{au}}{a} + C.$

14. $\int ue^{au}\,du = \dfrac{e^{au}}{a^2}(au-1) + C.$

15. $\int u^n e^{au}\,du = \dfrac{u^n e^{au}}{a} - \dfrac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\int u^n \ln u\,du = \dfrac{u^{n+1}\ln u}{n+1} - \dfrac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (a) (6 pts) Find the Gini coefficient of inequality for the nation whose Lorenz curve of income distribution is given by $y = f(x) = 0.3x^3 + 0.5x^2 + 0.2x$.

(b) (3 pts) Find the average value of the function $g(x) = \dfrac{2x}{\sqrt{3x+4}}$ on the interval $[0, 4]$.

2. (9 pts) Find the consumers' and producers' surplus at equilibrium for the market with the following supply and demand equations

$$\text{Supply: } p = 10 + 2q \quad \text{and} \quad \text{Demand: } p = 110 - \frac{q^2}{80}.$$

3. A household's utility function is given by

$$U(x, y, z) = 7 \ln x + 10 \ln y + 18 \ln z,$$

where x, y and z are the quantities of goods of type X, Y and Z, consumed by the household each month, respectively. The prices per unit for these three goods are $p_x = \$5$, $p_y = \$10$ and $p_z = \$25$, respectively.

(a) (6 pts) Find the quantities x^*, y^* and z^* of these goods that should be consumed each month to maximize the household's utility, given that their monthly XYZ-budget is $B = \$7000$.

(b) (3 pts) Use the envelope theorem and linear approximation to estimate the change in the household's maximum utility if p_z increases from \$25 to \$26.50, assuming that nothing else changes.

4. (9 pts) ACME Widgets produces and sells two competing goods. The demand equations for these goods are

$$q_1 = 100(5 - p_1 + p_2) \quad \text{and} \quad q_2 = 100(6 + p_1 - 2p_2),$$

where q_1 and q_2 are the daily demands for Type 1 widgets and Type 2 widgets respectively, and p_1 and p_2 are their respective prices. ACME's daily cost function is

$$c(q_1, q_2) = 5q_1 + 4q_2 + 500.$$

Find the prices that ACME should charge for their goods to maximize their daily profit. Find the corresponding daily demands for these goods and the maximum daily profit. Use the second derivative test to verify that the critical profit you found is indeed a *maximum*.

5. The average monthly demand (q, measured in 1000s of units) for a monopolistic firm's product is related to the price of their product (p, measured in dollars) and the average monthly household income in the market for the firm's product (y, measured in \$1000s), by the equation:

$$q = \frac{20(5y-4)^{1/2}}{3p+1}.$$

(a) (5 pts) Compute q, $\partial q/\partial p$ and $\partial q/\partial y$ when $p = 8$, and $y = 4$.

(b) (2 pts) What is the income-elasticity of demand when $p = 8$ and $y = 4$?

(c) (2 pts) Use your answer to (b) to estimate the ***percentage*** change in demand, if income decreases to $y = 3.8$. What assumption do you need to make to justify this estimate?

6. (9 pts) The production function for ACME WIDGETS is given by

$$Q = 25K^{0.6}L^{0.4},$$

where Q is the number of widgets ACME produces in one year, K is the number of units of capital input and L is the number of units of labor input ACME uses to produce their widgets. The price per unit of capital input is $p_K = \$4,000$ and the price per unit of labor input is $p_L = \$2000$.

Find the levels capital and labor input that ***minimize the cost*** of producing $q = 1,000$ widgets.

What is the corresponding minimum cost? What is the marginal cost at this level of production?