Final Exam

Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided.
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Take your time. Answer each question completely. Check your answers.***
- ***For full credit—explain/show your work.***

Good Luck!!!

NAME: ______________________________

Problem	Score
1	/9
2	/9
3	/9
4	/9
5	/9
6	/9
Total	/50

Selected Integration Formulas

Basic rules.

1. $\displaystyle\int u^k\,du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C.$

3. $\displaystyle\int e^u\,du = e^u + C.$

4. $\displaystyle\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\displaystyle\int c\cdot f(u)\,du = c\cdot\int f(u)\,du.$

Rational forms containing $(\mathbf{a}+\mathbf{bu})$.

6. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C.$

7. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C.$

8. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C.$

9. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a}+\mathbf{bu}}$.

10. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle\int e^{au}\,du = \frac{e^{au}}{a} + C.$

14. $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C.$

15. $\displaystyle\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\displaystyle\int u^n\ln u\,du = \frac{u^{n+1}\ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (9 pts) The marginal propensity to save for a small nation is given by

$$\frac{dS}{dY} = \frac{2Y+1}{12Y+13},$$

where both national savings S and national income Y are measured in billions of dollars.

Find the total change in the nation's savings and consumption if income increases from \$10 billion to \$20 billion.

2. (9 pts) Find the consumers' and producers' surplus at equilibrium for the market with the following supply and demand equations

$$\text{Supply: } p = 5 + q^2/40 \quad \text{and} \quad \text{Demand: } p = 125 - 0.5q.$$

3. A household's utility function is given by

$$U(x, y, z) = 15\ln x + 6\ln y + 4\ln z,$$

where x, y and z are the quantities of *Xidgets*, *Yidgets* and *Zidgets*, respectively, consumed by the household each month. The prices per unit for these three goods are $p_x = \$20$, $p_y = \$10$ and $p_z = \$5$, respectively.

(a) (6 pts) Find the quantities of Xidgets, Yidgets and Zidgets that should be consumed each month to maximize the household's utility, given that their monthly XYZ-budget is $B = \$4000$.

(b) (3 pts) By approximately how much will the household have to increase their monthly XYZ-budget from its current level to increase their (maximum) utility by 3 utils? Explain your answer briefly.

4. (9 pts) Find the critical point(s) and critical value(s) of the function

$$h(u,v) = u^2 - 2uv + \frac{1}{3}v^3 - 8v + 2$$

and classify the critical value(s) as relative minima, relative maxima or neither using the second derivative test.

5. The average monthly demand (q, measured in 1000s of units) for a monopolistic firm's product is related to the price of their product (p, measured in dollars) and the average monthly household income in the market for the firm's product (y, measured in \$1000s), by the equation:

$$q = \frac{30\sqrt{3y+4}}{2p+3}.$$

(a) (5 pts) Compute q, $\partial q/\partial p$ and $\partial q/\partial y$ when $p = 6$, and $y = 4$.

(b) (2 pts) What is the income-elasticity of demand when $p = 6$ and $y = 4$?

(c) (2 pts) Use your answer to (b) to estimate the ***percentage*** change in demand, if income increases to $y = 4.3$? What assumption do you need to make to justify this estimate?

6. The production function for ACME WIDGETS is given by

$$Q = 30K^{0.6}L^{0.4},$$

where Q is the number of widgets ACME produces in one year, K is the number of units of capital input and L is the number of units of labor input ACME uses to produce their widgets.

The price per unit of capital input is $p_K = \$6,000$ and the price per unit of labor input is $p_L = \$2500$.

(a) (6 pts) Find the levels capital and labor input that ***minimize the cost*** of producing $q = 2,000$ widgets. What is the corresponding minimum cost?

(b) (3 pts) Suppose that the parameter $\alpha = 30$ in the production function increases to $\alpha_1 = 32$, because of technological improvements in the production process. Use the envelope theorem and linear approximation to estimate the resulting change in the minimal cost of producing $2,000$ widgets. Show/explain your work.