Review Questions 5

Table of integrals and differential equations

1. Compute the following integrals

a. $\displaystyle\int 5x\sqrt{7-2x}\,dx =$

b. $\displaystyle\int \frac{7t^2+3t-1}{2+5t}\,dt =$

c. $\displaystyle\int 500t^2e^{-0.04t}\,dt =$

d. $\displaystyle\int \frac{3e^{2x}}{\sqrt{4+e^x}}\,dx =$

e. $\displaystyle\int \frac{300}{1+0.25e^{-0.1t}}\,dt =$

f. $\displaystyle\int \frac{4(\ln x)^2}{3x\sqrt{2+7\ln x}}\,dx =$

2. Let $y = f(x)$ satisfy (i) $\dfrac{dy}{dx} = 3xy^2$ and (ii) $y(1) = 2$. Find the function $f(x)$.

3. The income-elasticity of demand for a firm's product is proportional to the square root of income. Find the demand as a function of income, given that $q(100) = 50$ and $q(400) = 90$.

4. The population of a tropical island grows at a rate that is proportional to the *third root* ($\sqrt[3]{\ }$) of its size. In 1950, the islands population was 1728 and in 1980, the islands population was 2744. What will the Islands population be in 2020?

5. The population of bass in a large lake grows according to the (logistic) model,

$$\frac{dY}{dt} = 0.05Y(10-Y),$$

where Y is the size of the bass population, measured in 1000s of fish, and t is measured in years. (I.e., if the population is 2000, then $Y = 2$.)

(a) If the bass population in 1990 was 1500, what will the population be in 2010?

(b) When will/did the bass population reach 5000?

(c) Once the population reaches 3000, bass are 'harvested' from the lake at the constant rate of 1000 fish per year. Describe what will happen to the fish population over time.