Review Questions 6

Partial derivatives and applications.

1. Compute the indicated partial derivatives of the functions below.

a. $z = 3x^2 + 4xy - 5y^2 - 4x + 7y - 2,$

$z_x =$

$z_y =$

b. $F(u, v, w) = 60u^{2/3}v^{1/6}w^{1/2}$

$\dfrac{\partial F}{\partial u} =$

$\dfrac{\partial^2 F}{\partial w \partial u} =$

c. $w = x^2 z \ln(y^2 + z^3)$

$w_x =$

$w_y =$

$w_{xx} =$

$w_{yz} =$

$w_{xyz} =$

d. $f(x, y, z) = 2x^3yz^2 - 3xy^3z + 5x^2y^2 - 7yz^5 + 11x - 1$

$f_x =$

$f_y =$

$f_{zx} =.$

e. $q(u, v) = \dfrac{u^2v - 3uv^3}{2u + 3v}$

$\dfrac{\partial q}{\partial u} =$

$\dfrac{\partial q}{\partial v} =$

2. The monthly cost function for ACME Widgets is

$$C = 0.02Q_A^2 + 0.01Q_AQ_B + 0.03Q_B^2 + 35Q_A + 28Q_B + 5000,$$

where Q_A and Q_B are the monthly outputs of type A widgets and type B widgets, respectively, measured in 100's, (so, for example, if 3000 type A widgets are produced in a month, then $Q_A = 3000/100 = 30$). The cost is measured on dollars.

a. Compute the marginal cost of type A widgets and the marginal cost of type B widgets, if the monthly outputs are 25000 type A widgets and 36000 type B widgets.

b. Suppose that production of type A widgets is held fixed at 25000, and production of type B widgets is increased from 36000 to 36050. Use your answer to part a. to estimate the change in cost to the firm.

c. Suppose that production of type A widgets is increased from 25000 to 25060, and production of type B widgets is increased from 36000 to 36040. Use your answer to part a. to estimate the change in cost to the firm.

3. The annual production function for SlugTools Inc. is

$$Q = 2.4K^{1/3}L^{3/5} + 3K^{1/2}L^{1/4},$$

where K and L are the capital and labor inputs, respectively, and where Q, K and L are all measured in \$1000's

a. Compute the marginal products of capital and labor, when capital input is \$200000 and labor input is \$320000.

Remember: The marginal products of capital and labor are Q_K and Q_L, respectively.

b. Compute the capital and labor elasticities of output at the same input levels.

c. Suppose that capital input is held fixed at \$200000, and labor input is raised from \$320000 to \$325000. Use your answer to part b. to estimate the **percentage change** in output.

4. The demand function for a monopolist firm's product is given by

$$Q = \frac{Y_d^{1/3}(2P_s + 5)^{1/2}}{P + 1},$$

where

- Q is the monthly demand for the firm's product, measured in 1000's of units;
- P is the price per unit of the firm's product, in dollars;
- P_s is the average price per unit of substitutes for the firm's product;
- and Y_d is the average monthly household disposable income in the market for the firm's product, in dollars.

a. Find Q_P, Q_{P_s} and Q_{Y_d}.

b. Compute the *income-elasticity* of demand for the firm's product when $P = \$5.25$, $P_s = \$4.95$ and $Y_d = \$2800.00$.

c. Compute the *price-elasticity* of demand when $P = \$5.25$, $P_s = \$4.95$ and $Y_d = \$2800.00$.

d. The government passes a middle-class tax-cut bill that will *raise* average household disposable income by 5%, and the firm raises the price of its product to \$5.30.

Use your answers to b. and c. to compute the approximate *percentage* change in monthly demand for the firm's product these changes cause, assuming that the average price of a substitute does not change.

5. Suppose that $z = 3x^2y^3 + 5xy^2 - 3x + y - 1$, where $x = 3t - 2$ and $y = 2t + 1$.

Use the ***chain rule*** to compute $\left.\dfrac{dz}{dt}\right|_{t=0}$.

6. The production function for a firm is given by

$$Q = F(K, L),$$

where Q is the firm's monthly output, K is the firm's monthly capital input and L is the firm's monthly labor input. Furthermore, the labor input L is given by

$$L = 2m(60h - h^2),$$

where m is the number of the firm's employees and h is the average number of hours each employee works per month. The firm's profit function is given by

$$P = p_0Q - (w_0mh + K),$$

where p_0 is the (fixed) price of the firm's product and w_0 is the (average) hourly wage the firm pays its employees.

Assuming that K is *independent* of m and h, use the ***chain rule*** to find $\dfrac{dP}{dm}$ and $\dfrac{dP}{dh}$. Express your answers in terms of F_L and F_K, as needed.