

Review Questions 7

Optimization in several variables, I

1. Find the *quadratic Taylor polynomial* for the functions below, centered at the indicated points.

(a) $f(x, y) = \sqrt{5x + 2y}$, centered at the point $(x_0, y_0) = (10, 7)$.

(b) $g(u, v) = \ln(u^2 + 3v)$, centered at the point $(u_0, v_0) = (0.5, 0.25)$.

2. Find the critical points and critical values of the functions below.

a. $f(x, y) = 3x^2 - 12xy + 19y^2 - 2x - 4y + 5$.

b. $g(s, t) = s^3 + 3t^2 + 12st + 2$.

c. $h(u, v) = u^3 + v^3 - 3u^2 - 3v + 5$.

3. Consider the function $F(x, y) = 3x^2 - Axy + By^2 - 2x - 4y + 5$, with *variables* x and y and *parameters* A and B .

(a) Find the critical point and critical value of this function when $A = A_0 = 12$ and $B = B_0 = 19$.

(b) Use the *envelope theorem* and *linear approximation* to predict how the critical value will change if the parameter A changes from $A_0 = 12$ to $A_1 = 12.5$ and the parameter B changes from $B_0 = 19$ to $B_1 = 19.2$.

4. Use the second derivative test to classify the critical values that you found in problem 2.

5. ACME Widgets produces two competing products, type A widgets and type B widgets. The joint demand functions for these products are

$$Q_A = 100 - 3P_A + 2P_B \quad \text{and} \quad Q_B = 60 + 2P_A - 2P_B$$

and ACME's cost function is

$$C = 20Q_A + 30Q_B + 1200.$$

Find the prices that ACME should charge to maximize their profit. Justify your claim that the prices you found yield the absolute maximum profit.

6. Find the critical point(s) of the functions below. You do not need to classify the critical values in this problem.

- a. $H(u, v, w) = 2u^2 + v^2 - 3w^2 + 2uv + 4uw - 2vw$.
- b. $F(x, y, z) = 30x^{1/3}y^{2/3} - z(5x + 8y - 400)$.
- c. $G(w, x, y, z) = x^2 + 2y^2 + 4z^2 - 2wx - 5wy - 3wz + 300w$.

7. A firm sells two competing products, A and B. The joint demand equations for these products are

$$Q_A = 80 - \frac{3}{2}P_A + 2P_B \quad \text{and} \quad Q_B = 60 + 2P_A - 3P_B,$$

where Q_A and Q_B are the weekly demand for products A and B, respectively, and P_A and P_B are the prices of these products.

Find the prices that the firm should charge to maximize the firm's *revenue*, the quantities that the firm sells at these prices and the (maximum) revenue the firm generates.

Justify your claim that the prices you found yield the maximum revenue.