Review Questions 8

Optimization in several variables, II

1. Find the optimal values for the constrained optimization problems below. You may assume that (one of) the critical point(s) you find yield(s) the desired optimum.

  a. Find the minimum value of $f(x, y) = x^2 + y^2$ subject to $3x + 5y = 68$.

  b. Find the maximum value of $g(x, y, z) = 20x^{1/2}y^{1/3}z^{1/6}$, subject to the constraint $5x + 4y + 7z = 1680$.

  c. Find the maximum and minimum values of the function $h(x, y) = 3x + 5y$ subject to the constraint $x^2 + y^2 = 136$.

2. Jack's (gustatory) utility function is $U(x, y, z) = 5\ln x + 7\ln y + 18\ln z$, where x is the number of fast-food meals Jack consumes in a month; y is the number of 'diner' meals he consumes in a month; and z is the number of 'fancy restaurant' meals he consumes in a month.

The average price of a fast-food meal is $p_x = \$4.00$; the average price of a 'diner' meal is $p_y = \$8.00$; and the average price of a 'fancy restaurant' meal is $p_z = \$30.00$.

  a. How many meals of each type should Jack consumer per month to maximize his utility, if his monthly budget for these meals is $\beta = \$1200.00$?

  b. By approximately how much will Jack's utility increase if his budget increases by $50.00? Explain your answer.

3. The production function for SlugTools Drills, Inc. is $P(K, L) = 10K^{2/5}L^{3/5}$, where $P(K, L)$ is the number of drills produced in a year, K is the capital input, and L is the labor input. The cost per unit of capital is $1280, and the cost per unit of labor is $14580.

  a. Find the levels of labor and capital inputs that **minimize** the cost of producing 20480 drills. You may assume that the critical point you find does in fact yield the minimal cost.

  b. Find the levels of labor and capital inputs that **minimize** the cost of producing q drills. Express your answer in terms of q.

  c. Use your answer to part b. to find the **marginal cost** function for SlugTools.

  d. Assuming that the fixed cost for SlugTools is $C_0 = \$100,000$, find their cost function.

4. Consider the firm in RQ #7, problem 5. The firm's cost function there is

$$C = 20Q_A + 30Q_B + 1200,$$

so the marginal cost (to the firm) of product A is $C_{Q_A} = 20$.

Use the *envelope theorem* and linear approximation to estimate the change in the firm's profit if the marginal cost of product A increases from 20 to 21, assuming that the demand equations and all the other parameters remain the same (as in RQ 7, #5).

Comment: The computational aspect of this problem is easy, once you set it up correctly.