Review Questions 9

Final Review

1. The annual output for a discount hotel chain is given by $Q = 30K^{2/5}L^{1/2}R^{1/4}$, where K, L and R are the capital, labor and real estate inputs, all measured in $\$1,000,000$s, and Q is measured in rooms rented.

    a. What is the annual output when the inputs are $K = 20$, $L = 15$ and $R = 5$?

    b. Use *linear approximation* to estimate the change in output if if capital input remains fixed, labor input increases by $\$750,000$ and real estate input increases by $\$500,000$?

2. The hotel chain from problem **1.** has an annual budget of $B = \$69$ million.

    a. How should they allocate this budget to the three inputs in order to *maximize* their annual output? What is the maximum output?

    b. What is the critical value of the multiplier when output is maximized?

    c. Use your answer to **b.** to compute the *approximate* change in the firm's maximum output if their annual budget increases by $\$500,000$? Explain your answer.

3. The price elasticity of demand for a monopolistic firm's product is given by

$$\eta_{q/p} = -0.1\sqrt{p}.$$

When $p = 16$ the demand is $q(16) = 200$, what is the demand when $p = 25$?

4. Find the consumers' surplus and producers' surplus for the market whose supply and demand equations are given below:

$$
\begin{aligned}
\text{Supply} &— & p &= 0.01q^2 + 5 \\
\text{Demand} &— & p &= 60 - 0.6q
\end{aligned}
$$

5. A consumer's utility function is given by $U = 3\ln(x+2) + 2\ln(y+3) + \ln(z+5)$, where x, y and z are the monthly quantities of food, entertainment and clothing that the consumer purchases. The (average) price per unit of food is $p_x = \$10.00$, the (average) price per unit of entertainment is $p_y = \$20.00$, and the (average) price per unit of clothing is $p_z = \$25.00$.

    a. If the consumer's monthly budget for food, entertainment and clothing is $\$1500.00$, then how much should she spend on each good to maximize her utility? What is the maximum utility that she achieves? Round your answers to the nearest whole numbers.

    b. By (approximately) how much would the consumer need to increase her monthly budget to increase her (maximum) utility by one unit?

6. Compute the present value of an annuity that pays at the annual rate $f(t) = 200t^2$ for $T = 10$ years, if the interest rate is $r = 4.5\%$, compounded continuously.

7. A monopolistic firm sells one product in two markets, A and B. The daily demand equation for the firm's product in each market are given by

$$Q_A = 100 - 0.4P_A \qquad \text{and} \qquad Q_B = 120 - 0.5P_B,$$

where Q_A and Q_B are the demands and P_A and P_B are the prices for the firm's product in each market, respectively. The firm's constant marginal cost is \$40 and the its daily fixed cost is \$2500.

a. Find the prices that the firm should charge in each market to maximize its daily profit. Use the second derivative test to verify that the prices you found yield the *absolute* maximum profit.

b. The governments imposes a 10% sales tax on the firm's product (in both markets). Find the government's daily revenue from the tax and analyze the cost of the tax to the firm and to consumers in each market. Specifically, you should

   (i) find the firm's new (maximum) daily profit,
   (ii) find the new (total) price to consumers in each market, and
   (iii) discuss the results.

8. Compute the Gini coefficient (of inequality) for a country whose Lorenz curve (of income distribution) is given by

$$y = \frac{10^x - 1}{9}.$$

(See problem 35 in section 14.10.)

9. A firm's marginal revenue and marginal cost functions are given by

$$\frac{dr}{dq} = 0.4q\sqrt{400 - 0.64q^2} \qquad \text{and} \qquad \frac{dc}{dq} = 0.2q + 25.$$

Express as a **definite integral** the **total change** in the firm's **profit** when it increases output from $q_1 = 10$ to $q_2 = 20$, and compute the integral.